Non-existence of Irreversible Processes in Compact Space Time

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Abstract

It is shown that if physical space time were truly compact there would only be of the order of one solutions to the classical field equations with a weighting to be explained. But that would not allow any peculiar choice of initial conditions that could support a non-trivial second law of thermodynamics. We present a no-go theorem: Irreversible processes would be extremely unlikely to occur for the almost unique solution for the intrinsically compact space time world, although irreversible processes are well known to occur in general. What we here assume – compact space time – excludes that universe could exist eternally. In other word if universe stays on forever (i.e. non-compact in time direction) our no-go theorem is not applicable.

PACS numbers: 98.80.Cq, 95.30.Tg, 98.80.-K

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I. INTRODUCTION

It has been suggested by Hartle and Hawking that the initial state of the universe – the wave function of the universe – be given by the "no boundary postulate". [1] The idea is that the wave function be given by a functional integral over the field configurations on a space time manifold. If the model is used only to give a wave function there is made an assumption about a kind of past, but no assumption about the future that also ends without boundary. It is not in such a wave function for the universe formulation assumed should be valid for the future development. It would, however, seem natural from the spirit of the no boundary assumption that it should be valid for both future and past. We have also recently discussed a model with in a slightly special way of compactified space time – or rather just time – [3, 4, 5, 6, 7], in which the time axis be an S^1-circle. In these articles we discussed the possibility of the time axis with S^1-topology, which is equivalent to consider a world with an intrinsic periodicity. The major problem for such a model which we found was that entropy would be constant as a function of time, i.e. there would be no irreversible processes in such a universe. Our main concern in the present article is an extension of this result:

Provided the space time is <u>compact</u> there cannot be irreversible processes, in the classical approximation.

The main point of the present article is indeed that for a compact space time the number of solutions to the classical equations of motion becomes essentially of order one after an appropriate discretization, or weighted in a way to be discussed below. Then there is no place for making more assumptions about the special solution to the equations of motion selected by Nature. Thus there is no more any possibility for specifying the initial state which leads for us to present a <u>no-go theorem</u> that will be discussed in detail in section 5 in the present article. But such an initial state specification is highly needed to implement the second law of thermodynamics in a non-trivial way. By a non-trivial implementation we here mean one involving irreversible processes.

In the following section 2, we shall seek as to how we think about the very general classical field theories discussed here in a concrete manner, and how we cut them off, both by a lattice in space time and by a lattice in the value space for the fields (components). We allude to our philosophy that the equations are sufficiently complicated such that we

can use statistical arguments in discussing them. In section 3 we then discuss a few aspects of estimate of the number of solutions for a compact space time, but we seek also to put the effect of non-compact corners of the space time manifold in perspective. In section 4 we introduce the concept of macro state developments so as to be able to discuss entropy and what an irreversible process involves.

Then in section 5 we complete the argument that irreversible processes are extremely unlikely to occur for the "essentially" unique solution to the equations of motion. In section 6 we conclude and put forward some outlook as to how to get irreversible processes at all compatible with a sensible micro physics cosmology.

II. GENERAL CLASSICAL FIELD THEORY AND CUTTING IT OFF

It is the spirit of the present article to perform an extremely abstract and general discussion about a general classical field theory with fields defined as functions on a space time manifold as in general relativity. The main interest in this article is to make a no go theorem for the compact space time manifold, but at first we can set up our formalism and thinking even for manifolds that are not compact.

To avoid a lot of technical problems and details in connection with Lyapunov exponents [11] which anyway turn out to cancel, we shall work with a regularized or cut off theory. Since we work only classically, the usual type of cut off on quantum field theory will, however, not be needed. Nevertheless we shall consider two types of cut off:

1) We shall think of a latticification of space time by letting the classical fields be defined only on a set of sites. Since the manifold has a priori even a complicated topology – at least we should be able to have a compact manifold – we cannot have a completely regular lattice on the manifold, but that is presumably not needed either for the very abstract discussions in the present article.

2) We shall also discretize the set of values that the assumed multicomponent field can take on.

After making these two cut off procedures our general classical field theory has been converted into a set of "lattice" point on the manifold on which multicomponent fields are defined that take values in a discrete value space.

The equations of motion will be approximated by relations between the field values on a bunch of a few neighboring sites.

If we imagined that at the continuum field level we introduced enough conjugate momentum fields so as to make the field equations become first order partial differential equations, a naive discretization would lead to the form that we could write the field equations as a relation between the field components on a bunch of $d+1$ neighboring sites. Here d is the dimension of the space time manifold so that in the physically relevant case this dimension would be $d = 4$. Thus we should have relations for bunches of 5 neighboring points. Since we are mainly interested in counting equations and degrees of freedom it is very important that we shall have just one such relation between a bunch of $d+1$ lattice points for each lattice point. That is to say we shall have in the bulk of the manifold just equally many relations as field components on the sites. More precisely, if we call the number of field components for the field ϕ, say n, then there are n field components per site. Since there should be also n equations – i.e. an n-component relation there should be just equally many relations as variables in the bulk. This is what is naively taken a discretized solution set.

In cut off language the statement of the discretized solutions looses its interest because any solution is anyway discretized. We have however still some meaning attached to the concept analogous to the fact that there are equally many equations as variables: We have a disposition set F for simplicity, with a finite number of values l; cardinal number of F $= ktF$ for the multicomponent field at a site. If each of the n components could take k values, $l = k^n$. Then a $d+1$ site involving relation R is a subset of $\underbrace{F \times \cdots \times F}_{d+1} = F^{d+1}$. If this relation corresponds to an equation of the deterministic type in accordance to the equations of motion, we should require that for given values y_i $(i \neq j)$ of all but one in an ordered set $(y_1, y_2, \cdots, y_{j-1}, x, y_{j+1}, \cdots, y_{d+1}) \in R$ the component x sitting on the j-th place is uniquely determined from the requirement of the full ordered set of $d+1$ components belonging to the relation R. We both require that such an x exists and that there be just one x for any choice of the $(d+1) - 1 = d$ other components. In this case the cardinal number for the relation R is just $\frac{(ktF)^{d+1}}{ktF} = (ktF)^d = l^d$. For the just described type of relation between the $d+1$ sites we say that it reflects one equation.

It is the view point of the present article that the equations of motion are so complicated and have so many parameters – coupling constants etc. – that we effectively consider them

random. There is though the exception that we shall below assume that the random elements in the equations of motion do not mix the different macro states, when we study macro states and macro restrictions in order to be able to discuss the concept of irreversible processes. Return we shall to this exception in the randomness below in section 4.

Inside the macro restrictions imposed we can consider the relations as random. That is to say that we in principle let R be chosen as a random one among all those obeying the properties described from the macro description requirements.

Such considerations are, however, only used in the mild form in such the case as we use them as an excuse to simply make statistical considerations to estimate e.g. numbers of solutions. But it should be kept in mind that such estimations will work in almost all cases and that it is thus a very mild assumption to the validity of such arguments.

III.   NUMBER OF SOLUTIONS

In the case in which there are just equally many equations as variables as is the case for one field, i.e. the equations with the same number of components as the fields per lattice point, we have already seen that the solutions are discrete even when we do not discretize the field value space.

When we instead think of the discretized field value space we can use this coincidence of number of equations and variables to conclude that the number of solutions will at most be of order unity, if there are solutions at all. The argument is as follows:

If we have the number of lattice points $\sharp L$ on which the field ϕ is defined and takes values in a disposition space F with $l = \sharp F = ktF$ elements, there exist a priori $l^{\sharp L}$ different field functions. Here kt denotes cardinal number of F. We impose each time the field equation relation involving as above described $d + 1$ lattice points and only keep those field configurations satisfying the imposed relation, we reduce the number of field configurations by a factor l. The relation R only allow one out of l configurations on the $d + 1$ sites involved. After imposing $\sharp L$ equations, which means one per site we thus get reduced the original number of field functions on the lattice $l^{\sharp L}$ by a factor $l^{\sharp L}$ leaving a number of solutions of the order of $\frac{l^{\sharp L}}{l^{\sharp L}} \simeq 1$. This means that it could easily be zero, i.e. no solutions at all, or it could be few. It could, however, not likely be a huge number unless there is some systematic regularity, which is really what we thought to exclude by our assumptions in the

5

previous section 2.

This result is of major importance for the present article because it means that approximately and statistically there is just one solution to the equations of motion in latticized and in value or disposition space discretized model by assuming that when there is a finite number of lattice points as it corresponds to a compact space time.

If the lattice L is allowed to become infinite or if the value space is allowed to be infinite the above simple counting may no longer be trustable.

What could be especially importance would be if in some direction, typically in the $A \to \infty$ direction there occurs an infinite series of lattice points and if even say the universe expanded so that for cut off in time by only including $A \leq T$ where T is the infrared cut off in time, there become more and more lattice points on the infrared cut off border. In this case of the $A \leq T$ cut off with even expanding universe there will, for larger and larger T, be unavoidably more and more sets of the $d + 1$ lattice points associated with equation of motion which will be cut into pieces by the infrared cut off $A \leq T$. That is to say there will be more of these $d + 1$ local subsets of the lattice that have some of their elements with $A \leq T$ and some with $A > T$. These "cut to pieces" equations of motion elements (we may call such equations as equation of motion elements) can thus not be imposed on the infrared cut off theory. But then it means that they are lacking as equation of motion elements whereas so to speak the corresponding sites may well fall inside the included region $A \leq T$. It is easy to see that there is good reasons for expecting – statistically at least – that there will be more sites in the included region $A \leq T$ than fully included sets of $d + 1$ sites associated with equations of motion elements. This then means that the dimensionality of the solution space – in the continuum case – will go up and up with T proportional to the space volume of the universe at time T. The latter will be proportional to the number of cut $d + 1$ sets associated with the elements of equations of motion.

In the discretized or cut off language we will instead find that the number of solutions for the infrared cut off space time no longer is of order unity. Rather it goes as

$$l^{\sharp(L \cap \{t < T\})} - \sharp^{uncut\ equations}$$

$$\sim l^{constant \cdot \text{``Space vol at } t=T\text{''}} \tag{1}$$

Such a factor could be more significant than what we can obtain by uncertainties in the argument for the compact space time.

It should, however, be kept in mind that our level of intention with respect to the accuracy of the estimates for number of solutions is low. The reason is that we intend to compare the logarithms of such numbers with entropies measured in Boltzmann constant k unit. In this natural unit k the entropies are very large numbers – we could say Avogadro's number sizes – and the exponentials of them become even much more huge. If we only care for such an accuracy level every sensible number is of order unity.

We see that it is at least not excluded in the above arguments that a universe keeping to expand or just staying huge into an infinite future that needs an infrared cut off can cause molester of so many elements of equation of motion.

Thus our counting solutions to be essentially of order unity indeed depends strongly on compactness of the space time. In fact at least a big universe existing into an infinite future time axis $A \to \infty$ is excluded.

IV.   MACRO STATE, AND WHAT IS THE SIGNIFICANCE OF IRREVERSIBLE PROCESS?

A.   Why do we make classical thinking?

Since the main point of the present article is that the empirical occurrence of irreversible processes such as friction or heat conduction is not compatible with a classical field theory with compact space time in the setting introduced above, we need to put the concept of irreversible processes into our language. Usually one describes the processes which may possibly be irreversible in terms of thermodynamics states, which we may call macro states, meaning that they are states of whole macroscopic systems described by extremely fewer parameters than the ones needed for the micro degrees of freedom. That is to say we here take the point of view that corresponding to each or at least the most important fundamental states of the micro degrees of freedom there is a macro state. Typically there will be a huge number of different micro states that correspond to the same macro state. Usually one thinks of dimensions of the quantum mechanical Hilbert space, when one talks about the micro states. But we could equally well think about the micro state as a point in the phase space, and one could make a latticification of the phase space which simulates the Heisenberg uncertainty principle by taking the discrete points to approximately cover a

volume as required by this uncertainty principle. There are several – or rather two – different ways of thinking about the micro degrees of freedom as classical:

1) We could consider the micro world described by fields obeying Klein-Gordon equations and Maxwell equations. We could even think of Dirac or Weyl equations, but we would rather like to ignore the Fermion fields to avoid further complications in the present description of our ideas and make classical approximations for these fields. This is actually the method of making a classical description the most easily matching with the discussion in the previous sections.

2) An alternative and different way of making a classical theory approximation to the micro world is to describe the various elementary particles such as elections, protons, etc as classical particles in terms of their positions and momenta and with some their internal degrees of freedom. Especially describing the motion of molecules by such classical approximation may be a good approximation in some cases.

If one wants to be very precise one clearly needs to work quantum mechanically, but our main point is so abstract and our accuracy requirements so low requirement of accuracy that presumably even the crudeness of working classically may not matter for the accuracy sake.

However, philosophically it may open for the danger of throwing away our main point by clinging too much to quantum mechanics. In fact it may be tempting to invoke to an anthropic principle by using that we already know a lot empirically about that quantum state in which many world interpretation [8, 9, 10] have singled out a component of the wave function a lá Everett. This component could well be one making up at the end an extremely small part of the original Hartle-Hawking's wave function. By so doing one may thus soon end up working with a model that in reality only uses an extremely unlike probability part of the original Hartle-Hawking's wave function. This is a bit like assuming that humanity were created by miracle if just the existence of humanity selects a suppressed probability amplitude.

In the classical language which we prefer here to use we can more clearly see that if the number of solutions with the empirically correct behavior is much under unity the model truly speaking does not function. We should not believe that in a theory leaving zero solution behaves correctly compared to phenomenology.

Thus to not have all the problems about thinking objects or states only exist after they are observed, we prefer to use classical thinking which is a kind of niche for the present series of works. Then also the future will exist no matter whether we measure it or not and there will be a definite future calculable by means of the equations of motion from the past or in the case of a compactified space time even from the equations of motion alone!

B. What is an irreversible process

The irreversible processes are characterized by the entropy truly increasing with time, with strict inequality i.e. $\dot{S} > 0$. Here we suggested to use a definition of entropy as the logarithm of the number of micro states. Let us say we have put a lattice into the phase space and then the micro state in the above definition of entropy corresponds to the macro state which then the macro state contains the micro state and is thereby realized. Here the irreversible process means that the universe develops from a macro state with smaller entropy into the one with bigger. This development is in conformity with equations of motion. Generally classical equations of motion correspond actually to the unfolding as time goes on by a canonical transformation and as such conserves the phase space volume. That is to say that a low entropy macro state develops under the irreversible process into a macro state with a much bigger phase space volume. Hugely bigger because the phase space volume of the high entropy macro state is the exponential of this high entropy and entropies in general measured in the Boltzmann constant as unit are already very big numbers. But the phase space volume is conserved by the time development as is given by the fact that the equations of motion conserve the phase space volume because it were canonical transformation. We must understand that in the possible micro states often the irreversible process has taken place. The system can now be in is only a very tiny subvolume of the whole high entropy macro state phase space volume. To the discretization introduced there corresponds to the one of the phase space volume into points in phase space. In this cut off language the irreversible process implies then that there is only a very tiny number of micro states, though still huge relative to the even more huge total number of micro states in the high entropy macro state into which there is any chance for the system to go. The restriction to this tiny subset comes in because of the system having to come through the low entropy macro state. This means that only an exceedingly tiny part of the micro states in the high entropy macro

9

state are really unstable if they agree with the experiment or phenomenological observation of the irreversible process that took place and thus the solution – if there is only one as we derived in the compact space time case "essentially" – has to belong to this tiny subset.

V. THE CRUX OF OUR ARGUMENT: A COMPACT SPACE TIME UNIVERSE IS NOT PHENOMENOLOGICALLY VIABLE

A. Random part of Hamiltonian and irreversible process
– a no-go theorem –

In the view point that there exists a random part of the Hamiltonian at least as far as the moving around of the micro state inside its macro state we shall show in this subsection some conflict between the compact space time and the existence of irreversible processes. This random part of the Hamiltonian will be able to push around at random the relatively tiny subset of micro states inside the high entropy macro state, although we assume it restricted not to let the transitions between macro states be changed by the random part. This means that even if we thought at first that the essentially unique solution – in case of compact space time – belonged to this tiny subset, this property would be spoiled by a bit of change in the random part of the Hamiltonian. The main point is that in the random part of Hamiltonian it becomes exceedingly unlikely that the "essentially" unique solution should just be one of the tiny subset which could have come from the low entropy state by equation of motion.

We may make statement of the crux of the argument in the following:

By varying a bit randomly a part of the Hamiltonian which by itself are not causing transition between macro states, but they are caused by a past which is not considered random here, the "essentially" unique solution is varied naturally in the compact space time case. This random variation of the random part most importantly changes how the micro state is in detail in the era in which it passes through the macro state, we called the high entropy one. But now for the observed irreversible process to occur for the world developing accordingly to the "essentially" unique solution it is necessary that the latter during the passage of the high entropy macro state belongs to the tiny subset. But that is under the influence of the random Hamiltonian shuffling around exceedingly unlikely. This is the crux of our argument: It is exceedingly unlikely that the "essentially" unique solution will match

to this requirement from the observation of the irreversible process.

The conclusion from this then is that a compact space time is not phenomenologically viable! Logically of course we made a series of assumptions and approximations, but it is our point of view that the extra assumptions were very mild. By this mildness we mean that the content in them which were really relevant was correct and just healthy scientific judgment.

For instance our cut off procedures above are not reliable as to whether it really exists in nature – although we would not exclude the possibility that something close to that could be true – but we believe that we could with some mathematical effort reproduce the essential points by using either continuum classical physics or even quantum mechanics and quantum field theory. Also one should not take literally that we have a random contribution to the Hamiltonian. However, to really invalidate our argument by avoiding this random Hamiltonian philosophy would mean to have the solution do something that looked backward in time would be completely miraculous. To circumvent the random Hamiltonian part story we might simply bear in mind that the "essentially" unique solution was determined from the equations of motion which at least to high accuracy are time reversal invariant. The "essentially" unique solution thus carries no time arrow information. From the point of view of this solution backward and forward in time is quite on the same footing. So from this point of view it is equally miraculous that the brown mixed coffee and white cream should ever have been separated as if it suddenly separated. That is to say it would be a complete miracle if the "essentially" unique solution should lead to any irreversible processes as judged on the macro level. In fact we just have to invert the time axis and remember that with inverted time axis an irreversible process looks as a complete miracle caused by a completely unlikely micro configuration.

In this way we bring down our argument to just saying that the "essentially" unique solution is exceedingly unlikely to belong to the so seldom class that it is – time reversely looking – a complete miracle. Compact space time gives so may equations of motion that the solution becomes "essentially" unique. That is, however, incompatible with that the solution realized in nature with its irreversible processes, is of a so miraculously seldom type, that we need the solution to have been chosen with such miracles (=the irreversible processes) on purpose.

One cannot get irreversible processes by accident, rather only by tuning initial conditions

just for that purpose. Therefore the compact space time is not allowed to fix the solution so that one (=God) cannot fix it further so as to get the "miraculous" irreversible processes.

B. Configurations for which equations of motion progress a chain

In order to be able to apply our assumption of "no miracles in either way" the easiest is to get hold of a configuration really a co-dimension one series of lattice points the field values at which one can deduce the field values at another similar configuration of lattice points. From the locality of the field equations which only involve neighboring points it is rather obvious that to have two hyper-lines or, curves or pieces of hyper-lines or curves able to predict field values for each other they should together form a closed loop, although they could possibly close at infinity. Also it is intuitively expected that the two hyper-curve pieces should be as straight as possible. Curving too much might get them correlated with themselves. [12]

It turned out as can be easily seen that drawing one of the hyper-curves with full drawn line —— and the other with dashed line - - - the figures (see Fig. 1) of such series of lattice points related by equation of motion can take the form:
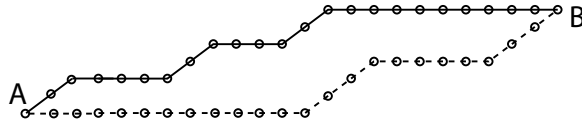

FIG. 1: Hyper-curve on a lattice representing equation of motion

We consider here the two outer points A and B belonging to both pieces of hyper-curves. In fact one can easily argue that if we consider the field value sets on the dashed curve, by means of the equation of motion, then we can successively get determined the field values inside the figure encircled by the two hyper-curve pieces - - - and —— and finally also get determined the field values along the hyper-curve drawn in uninterrupted manner. Presumably it is the easiest to illustrate that one can in principle calculate the mentioned field values by an example – a parallelogram (Fig. 2)–.

Suppose that we know the field components on the sites along the interrupted curve (it is really here two lines with the angle 120° formed between them). Let us also notice the rules for calculability: Once we know two field component sets (i.e. the fields on two sites)
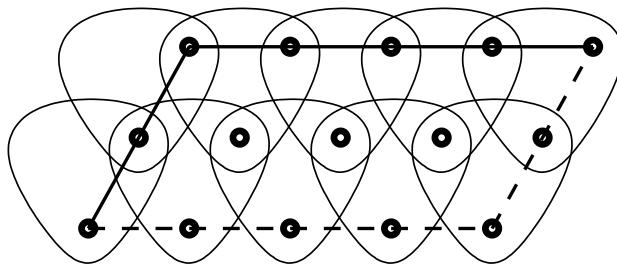
FIG. 2: Parallelogram related with field values. Inside triangles field components are calculable.

inside a set of three encircled sites by triangles $\triangle$ and $\triangledown$, then we can compute the field components at the third of those three sites uniquely.

Using this rule alone we can see, starting from the right most upgoing part of the interrupted line series of sites, that we can uniquely calculate the field components at the line just one lattice constant further to the left parallel with this line. Repeating the same argument a step further to the left after that and continuing stepwise one soon sees that we finally get in principle all the field components on all the sites in the encircled parallelogram determined uniquely by the two hyper-curve pieces. Especially the sites on the fully drawn curve piece get their field values determined.

It is also easy to see that, if one oppositely started by knowing the fields along the fully drawn line then one could determine all the fields in the parallelogram. This way one would, however, start by determining the fields along the horizontal line of lattice points just one lattice constant step below the uppermost side of the parallelogram.

We may conceive of these two hyper curves with interrupted and full drawn curves respectively as two space-like curves of general relativity in two dimensions representing two different moments of time. To make this way of thinking more obvious we could let the two pieces of hypercurves be continued in both directions but now following each other totally (see Fig. 3):

FIG. 3: Example of the case in which two hyper-curves are interrupted.

Such a total following means that we have added chains of lattice points to both hypercurves, the same lattice sites. Since clearly a point belonging to both is calculable from

knowing either of the hypercurves they do not play any role in the question of whether you can or cannot calculate the fields on one hypercurve from that on the other one.

But looking at it as two moments of time we can use the argumentation above that successive entropies had to be equal – i.e. entropy must be constant – in the one-dimensional case, under the assumption of "reversability".

If we by definition say that we shall use for the estimate of the entropy at one of these moments of time only the entropies along the hypercurve and the correlation entropy reductions between neighboring sites along the hypercurve, we get expressions like

$$\sum S_A + M_{A,\ A+1}. \tag{2}$$

Where A is along hypercurve. Here the M_{AB} are expressions for the reduction in entropy of the combined entropy of sites A and B due to their correlation.

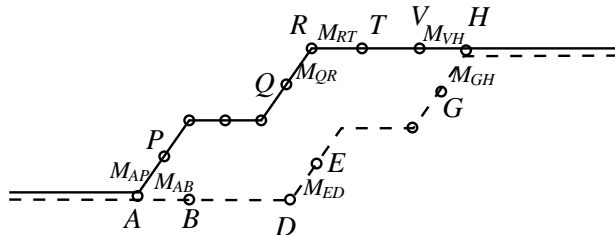In an example as Fig. 4 we get for the entropy corresponding to the interrupted piece of curve between A and H:

FIG. 4: Example of entropy connected with field along the hyper-curve.

$$\begin{aligned} S_{---} &= S_A + M_{AB} + S_B + \ldots + S_D + M_{DE} \\ &+ S_E + \ldots + S_G + M_{GH} + S_H \end{aligned} \tag{3}$$

This quantity of entropy is really the logarithm of the number of different states in which the fields on the sites from A to H along the interrupted line. This should be, as prescribed by the macro description, equal to the corresponding quantity for the fully drawn hypercurve by our "reversability in either" assumption. The expression for the logarithm of the number of micro states corresponding to the prescribed one is just given by

$$\begin{aligned} S\underline{\quad} &= S_A + M_{AP} + S_P + \ldots + S_Q + M_{QR} + S_R \\ &+ M_{RT} + S_T + \ldots + S_V + M_{VH} + S_H. \end{aligned} \tag{4}$$

14

A trivial mathematical trick to simplify the notation is to absorb the S-terms into the neighboring M_{AB}-terms by defining a quantity K_{AB} for every pair of neighboring sites on one of the two hypercurves considered

$$K_{AB} \ \hat{=} \ \frac{1}{2}S_A + \frac{1}{2}S_B + M_{AB}. \tag{5}$$

In this simplifying notation the equation $S_{---} = S\underline{\quad}$ or more precisely,

$$S_A + M_{AB} + S_B + \ldots + S_G + M_{GH} + S_H$$
$$= S_A + M_{AP} + S_P + \ldots + S_V + M_{VH} + S_H \tag{6}$$

becomes

$$K_{AB} + \ldots + K_{GH} = K_{AP} + \ldots + K_{VH}. \tag{7}$$

We can conceive of this latter equation as describing a flow of something – in fact entropy – crossing the links, AB say, in amount K_{AB}. Then the equation we derived tells this something is conserved.

Now, however, we have to discuss for which orientations of the used parallelogram we shall consider the truncated and full hyper-curves as tow different moments of times and thus should use our argumentation that the development should not be reversable either way.

C. Shall we assume no miraculous findings by moving in space?

Since we are anyway playing a mathematical game rather than working viable physics when we play the game of throwing away the second law, we may as well play several versions.

In the real world we are not surprised by high entropy density in some places than in others. It is quite non-miraculous that it is hotter at some distance away from some other place, or that there could be ice and water may be of the same temperature but of different entropy densities. In real nature with the second law of thermodynamics a temperature difference would not be obtainable in the long run and it would be smoothened out. This means that a temperature difference only in idealized models can be uphold from dismissing. Without second law and no miracles either we have only the one way out that there is

15

constant temperature throughout the universe. The temperature difference growing would be miraculous and thus should be excluded. In time reversed way the smoothing out of temperature would not be acceptable. Except for possibly several different phases realized in different regions we would thus expect already the thermodynamical equilibrium that does not allow temperature variations or chemical potential variations in space.

Provided that there are some conserved quantities it should, however, still be possible to have co-existing phases at the multiple point and thus it should <u>not</u> be necessary to have the same entropy density all through space . But for such a situation to be upheld some conserved charges $\partial_\mu j^\mu = 0$ would be needed.

At least with the possibility of having co-existing phases with all chemical potentials and the temperatures in balance at some separating border curve it seems that nothing should be miraculous locally at all. In as far as such a model should be realizable inside our very general scheme, one would expect that we should find that a scheme were realizable in our general scheme.

VI. CONCLUSION AND OUTLOOK

We have studied the possibility of having a compact space time and have come to the conclusion when using some very mild extra assumptions that a compact space is not compatible with the phenomenological fact that we have very often significant entropy increase, irreversible processes.

We have used classical approximation – and philosophy we could say – all through, but honestly speaking if we have to rely on quantum mechanics to get such numerically very big troubles as we estimate to go away, it sounds very suspicious to us: Since our main point is to count solutions, a passage through of the compact space time model by use of quantum mechanics would easily come under the suspicion of going through because one has forgotten that if you really get a very tiny overlap for quantum probability it can mean that there is really no overlap with the assumed wave function for the universe.

If one as it seems Hartle and Hawking do only use the wave function for the universe as it comes out of the functional integral without caring for if also the development into the future after the measurement has been done using such a wave function, then there is nothing that imposes a compact space time future.

Such a compact space time only to oneside in time is not in any trouble of the type caused by totally compact space time as we described it. What would be impossible is only if one wants also to have a no-boundary condition in the future.

It is really the problem that one gets equally many equations of motion as there are variables. This means that it is the problem to have irreversible processes in all models where there is not some places or times where due to infinities or singularities or for other reasons the number of equations compared to the number of variables gets reduced. But since we see the number of variables and the number of equations just matching very nicely where we have the phenomenological check, namely locally, it would be the most regular and simplest behavior if it continued like that.

A little possible addition to conclusion may be interesting to mention in the following: Since our argumentation is so strongly based on counting solutions there is also the possibility of escaping our conclusion of no irreversible processes even without disturbing the equality of the number of equations of motion and variables to be determined from them by the following loop hole:

If there were in some way made an enormous number of "attempts" to make a universe development – a "multiverse" theory – it could be o.k. even if one could just find one solution for one of these "attempts". In quantum mechanics in say the Feynman path way formulations there <u>is</u> in a formal way made such an enormous amount of "attempts". In a quantum mechanical theory one could thus possibly end up with a non zero but exceedingly small number of solutions per attempt and still get formally a sensibly looking result. From a classical way of thinking an equation system with in first approximation no solutions describing the world sounds ridiculous, but in a quantum formulation in which one always normalize distributions before one confronts them with experiment such a naively seen nonsense theory with an exceedingly unlikely universe may not be a true problem.

Acknowledgments

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[11] See e.g. ref.[7]

[12] The method to find number of solutions of differential equations in discretized space time is discussed in detail in [7] by the present authors.